A linear MmTSP formulation of robust location-routing problem: a dairy products supply chain case study

Javid Jouzdani* and Mohammad Fathian

School of Industrial Engineering,
Iran University of Science and Technology,
P.O. Box 163-16765, Narmak, Tehran, Iran
Fax: +98(21)89785535
Fax: +98(21)77209072
E-mail: javidjouzdani@iust.ac.ir
E-mail: fathian@iust.ac.ir
*Corresponding author

Abstract: The location-routing problem (LRP), which integrates location and routing decisions, has been an attraction to both researchers and practitioners. LRP as an important business problem has its own inherent uncertainty; especially, considering phenomena such as traffic congestion and weather conditions and their impact on transportation costs, routing in LRP is subject to uncertainty. In this paper, we propose a multi-depot multi-travelling salesman problem (MmTSP) formulation of robust LRP (RLRP) considering uncertainty in transportation costs. In addition, an appropriate technique is utilised to linearise the RLRP model. In order to justify the applicability of the proposed model, a dairy products supply chain case study is provided. Furthermore, comprehensive numerical examples are presented to characterise the performance of the proposed model.

Keywords: location-routing problem; LRP; multi-depot multi-travelling salesman problem; MnTSP; linearisation; robust optimisation; supply chain management.


Biographical notes: Javid Jouzdani is a PhD candidate in Industrial Engineering at the Iran University of Science and Technology. He received his MS in Socio-economic Systems Engineering from Amir Kabir University of Tehran (Polytechnic Tehran) and BS in Applied Mathematics from Isfahan University. He has also been an invited university lecturer in the past few years. His research interests are supply networks optimisation, analysis and modelling and its applications. He has published articles in journals such as Journal of Applied Mathematical Modelling, Journal of Uncertain Systems, International Journal of Operational Research and Journal of Applied Sciences and several other papers in international conferences.

Mohammad Fathian is an Associate Professor of Iran University of Science and Technology (IUST). He received his BS in the field of Electronic Engineering from KN Toosi University of Technology in 1991. He received his MS and PhD in the field of Industrial Engineering from Iran University of Science and Technology.
Technology in 1997 and 2002, respectively. He was the Head of the Department of Electronic Commerce of IUST from 2006 to 2011. He has published five books and more than 20 journal papers.

1 Introduction

In today’s competitive business world, companies have to compete under an increasing pressure of providing goods and services at lower prices with higher quality. Consequently, decision makers usually face multiple decision problems with several criteria. Considering all of the decision factors in decision making is practically burdensome if not impossible. Therefore, decision makers are selective about the factors they consider in their decisions. One of the key elements in running a successful business is an efficient and effective supply chain. A supply chain can be defined as the set of all the elements which are directly or indirectly involved in fulfilling the customers’ demands (Chopra and Meindel, 2007). A supply chain comprises of several subsystems including procurement, manufacturing, storage, transportation and retailing systems (Noorul Haq and Kannan, 2006). The concept of supply chain management (SCM) was introduced by Oliver and Webber (1982) as a response to the late ’70s quality revolution (Erenguc et al., 2006).

Supply chain decisions may be categorised into three levels: strategic, tactical and operational (Bender et al., 2002; Toyoglu et al., 2012). One of the most significant strategic decisions in SCM is location planning of the facilities involved in supply chain operations. On the other hand, as a tactical decision, routing of transportation vehicles has a major effect on the performance of the supply chain. Integration of different criteria in the aforementioned decision levels has been an important subject of research. Especially, integrating location and routing decisions has created an interesting area of research and practice known as location-routing problem (LRP).

Min et al. and Laporte presented the early reviews of the LRP (Wang et al., 2011; Laporte, 1989). In a more recent work, Nagy and Salhi reviewed the models, methods and issues of LRP (Nagy and Salhi, 2007). The applications of LRP include but are not confined to newspaper distribution (Jacobsen and Madsen, 1980), rubber plant location (Nambiar et al., 1989), military equipment location (Murty and Djang, 1999), optical network design (Lee et al., 2003) and shipping industry (Gunnarsson et al., 2005). For a more detailed list of LRP applications the reader is referred to a book chapter authored by Hassanzadeh et al. (2009).

In the past few decades, uncertainty in LRP has been a subject of research and application. Chan and Baker (2005) addressed the LRP with multiple depots, multiple vehicles and stochastically processed demands. In a research by Albareda-Sambola et al. (2007) a heuristic and lower bound for a stochastic LRP is proposed. The static conversion from brick-and-mortar retailing to click-and-mortar model is investigated from the perspective of distribution logistics in an article by Aksen and Altinkemer (2008). Nikbaksh and Zegordi (2010) designed a heuristic algorithm and obtained a lower bound for a two-echelon LRP with soft time window constraints. Yu et al. (2010)
proposed a simulated annealing heuristic for capacitated LRP. Boccia et al. (2011) utilised LRP concepts for designing a two-echelon freight distribution system. Fazel Zarandi et al. (2011) considered uncertain travel times as fuzzy numbers in a multi-depot capacitated LRP. LRP with simultaneous pickup and delivery is solved by using a branch and cut algorithm in an article by Karaoglan et al. (2011). In a similar research, Belenguer et al. (2011) proposed a branch and cut algorithm for LRP. Nadizadeh et al. (2011) proposed a greedy clustering method for capacitated LRP. Albareda-Sambola et al. (2012) proposed a multi-period LRP model with decoupled time scale to reflect the stability of location decisions as compared to routing decisions. By associating product amounts to the nodes of the network, a node-based product-flow (node-BPF) formulation approach for LRP is proposed by Toyoglu et al. (2012). A two-echelon LRP is solved by using a greedy randomised adaptive search procedure (GRASP), reinforced by a learning process and path relinking, by Nguyen et al. (2012). Escobar et al. (2013) proposed a two-phase hybrid heuristic algorithm for the capacitated LRP. Ting and Chen (2013) proposed a multiple ant colony optimisation algorithm for the capacitated LRP. In a separate work, Fazel Zarandi et al. (2013) addressed the capacitated LRP considering time windows and uncertainty and proposed a fuzzy chance constrained programming model. Using greedy clustering method, Zare Mehrjerdi and Nadizadeh (2013) solved the capacitated LRP with fuzzy demands.

LRP is consisted of two integrated problems: facility location problem (FLP) and vehicle routing problem (VRP). FLP is the problem of locating facilities such that total system cost is minimised. On the other hand, in VRP the goal is reducing transportation costs and improving customer service by selecting the optimal vehicle routes (Rajmohan and Shahabudeen, 2008). As a major issue in supply chain design, location decision are usually integrated with other decisions such as those related to inventory management (Diabat et al., 2009). Several formulations can be found for LRP in the literature. Regarding the routing, LRP formulations can be categorised as ‘vehicle-flow’ and ‘commodity-flow’ (Albareda-Sambola et al., 2007). The former only takes into account the circulations of vehicles and is closely related to travelling salesman problem (TSP) whereas the latter explicitly considers the quantity of goods transported through the network. Due to its importance in both research and practice, TSP has received a great deal of attention. A well-known generalisation of TSP is the multiple travelling salesman problem (mTSP). The mTSP addresses the routing of multiple salesmen departing from and returning to a single ‘depot’ node in the network. A comprehensive review of mTSP formulations and solution procedures is presented by Bektas (2006). As a special case of mTSP, single-depot multiple travelling salesman problem (SDmTSP) has been successfully applied to cell formation problem (Paydar et al., 2010). Considering multiple depots generalises the mTSP to multi-depot mTSP (MmTSP) in which there are multiple salesmen in each depot. In MmTSP, the salesmen may or may not return to the depot from which they depart. If the problem is to determine the routes for the salesmen who necessarily return to their original depots, the MmTSP is called a fixed destination MmTSP. By relaxing this constraint, the resulted problem is a non-fixed destination MmTSP. For a better understanding, some variants of the mTSP are depicted in Figure 1 (Jouzdani and Fathian, 2012). For a detailed formulation of TSP variants, one may refer to an article by Kara and Bektas (2006) and a more recent work by Jalali Naini et al. (2013).
Many published studies on uncertainty in LRP have modelled the uncertainty through fuzzy theory concepts (Fazel Zarandi et al., 2011; Escobar et al., 2013; Belenguer et al., 2011); robust decision making has also been successfully applied in SCM (Mohajer Tabrizi et al., 2012). We augment the non-fixed destination MmTSP model by incorporating a binary location decision variable. In addition, by utilising the robust optimisation concepts, proposed by Mulvey et al. (1995) and the techniques introduced by Yu and Li (2000), the proposed location-incorporated non-fixed destination model takes into account the uncertainty in the routing costs. Furthermore, interest rate is incorporated into calculations to consider the effect of time value of money on location and routing optimal decisions. More specifically, by using the time value of money, the future routing costs can be converted to their present value and then, added to the single payment of fixed facility location costs to constitute the total system cost. The main contributions of this paper can be summarised as follows:

1. Incorporating location decision into a non-fixed destination MmTSP to obtain an LRP model
2. Linearisation of the obtained model so that it can be solved in a reasonable amount of time by means of conventional methods such as Branch and Bound (B&B)
3. Considering uncertainty in transportation costs through concepts of robust optimisation
4. Incorporating time value of money (interest rate) into calculations
5. Justifying the applicability of the proposed model by providing a dairy supply chain case study.

The paper is organised as follows: in the next section, the mathematical model of the proposed RLRP is discussed. A case study of dairy products supply chain is presented in Section 3. In addition, Section 3 discusses the results obtained from the experiments with the numerical examples. Finally, Section 4 concludes the paper and presents some guidelines for future research.

2 The proposed RLRP model

In this section, the assumptions under which the model is developed and the formulation of the problem are presented and the linearisation method, utilised to linearise the non-linear RLRP, is discussed.
2.1 Assumption

The following assumptions are made in developing the proposed model:

1. The number of nodes in the network of the problem is known and fixed.
2. The fixed facility investment cost for each node is predetermined.
3. Any node in the network of the problem is a customer node. In addition, any node can also be a potential depot location.
4. If a depot is opened in a node, then the demand in that node is satisfied by the depot opened in the same node. Otherwise, the demand is satisfied by a depot opened in another node.
5. At most, one depot may be opened in each node.
6. The number of vehicles in each depot is bounded between $m_i$ and $M_i$.
7. The number of customers visited by each vehicle on its route is limited between a lower bound, $L$, and an upper bound, $M$. This assumption is made to consider the limitations imposed by constraints such as driver working hours and vehicle capacities.
8. The fleet of vehicles is homogenous, i.e., only one type of vehicle is used.
9. All model parameters are deterministic except the routing costs. The routing costs are known and subject to uncertainty expressed through probabilities under different scenarios.
10. The scenarios and the probability distributions of parameters are known.
11. Theoretically, the depots are opened to operate permanently. Practically, this translates to a long period of operation time.
12. The interest rate is known and constant, i.e., it has either no or ignorable future fluctuations.
13. The problem is considered as vehicle-flow LRP.

2.2 Mathematical formulation of the problem

In this paper, the network of the problem is represented by a graph denoted by $G = (V, E)$ where $V$ is the set of nodes and $E$ is the set of edges (links between nodes). In contrary to the original MmTSP model, each node in $V$ plays is both a customer node and a potential location for a depot. Suppose that $x_{ij}$ is a binary variable which equals 1 when the link from $i$ to $j$ is in the optimal solution and 0 otherwise. In addition, let $y_i$ be the location binary decision variable taking 1 when a depot is opened in node $i$ and 0 otherwise. Beside these decision variables, there are some model parameters which are introduced in Table 1.
Table 1: The nomenclature of model parameters and variables in order of appearance in the model.

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V$</td>
<td>The set of all nodes</td>
</tr>
<tr>
<td>$f_i$</td>
<td>The fixed cost of opening a depot in node $i$</td>
</tr>
<tr>
<td>$y_i$</td>
<td>Is 1 if a depot is opened in node $i$ and 0 otherwise</td>
</tr>
<tr>
<td>$r$</td>
<td>The interest rate</td>
</tr>
<tr>
<td>$\Omega$</td>
<td>The set of all scenarios</td>
</tr>
<tr>
<td>$p_\xi$</td>
<td>The probability of realisation for scenario $\xi$</td>
</tr>
<tr>
<td>$\psi_\xi$</td>
<td>Total routing cost under scenario $\xi$</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>The coefficient for deviation of the routing cost from its expected value</td>
</tr>
<tr>
<td>$\theta_\xi$</td>
<td>An auxiliary model variable used for linearisation</td>
</tr>
<tr>
<td>$E$</td>
<td>The set of all edges expressed as $(i,j) \in E$ representing the link from node $i$ to node $j$</td>
</tr>
<tr>
<td>$c_{ij}^\xi$</td>
<td>The routing cost from node $i$ to node $j$ under scenario $\xi$</td>
</tr>
<tr>
<td>$x_{ij}$</td>
<td>Is 1 if the edge from node $i$ to node $j$ is in the solution and 0 otherwise</td>
</tr>
<tr>
<td>$m_i$</td>
<td>Minimum number of salesmen in a depot (if opened) in node $i$</td>
</tr>
<tr>
<td>$M_i$</td>
<td>Maximum number of salesmen in a depot (if opened) in node $i$</td>
</tr>
<tr>
<td>$u_i$</td>
<td>A model variable that calculates the visit number of node $i$ used for sub-tour elimination</td>
</tr>
<tr>
<td>$L$</td>
<td>Minimum length for vehicle tours</td>
</tr>
<tr>
<td>$B$</td>
<td>A sufficiently large number</td>
</tr>
<tr>
<td>$K$</td>
<td>Maximum length for vehicle tours</td>
</tr>
<tr>
<td>$z_{ij}^k$</td>
<td>(For each $k$) is an auxiliary model variable used for linearisation</td>
</tr>
<tr>
<td>$\delta_k$</td>
<td>(For each $k$) is an auxiliary model parameter used for linearisation</td>
</tr>
</tbody>
</table>

Having these decision variables and model parameters, the following is the non-linear formulation for the proposed RLRP.

$$
\min z = \sum_{i \in V} f_i y_i + \frac{1}{r} \sum_{\xi \in \Omega} p_\xi \psi_\xi + \lambda \sum_{\xi \in \Omega} p_\xi \left[ (\psi_\xi - \sum_{\xi \in \Omega} p_\xi \psi_\xi) + 2\theta_\xi \right]
$$

Subject to

1. $$\psi_\xi = \sum_{(i,j) \in E} c_{ij}^\xi x_{ij}, \quad \forall \xi \in \Omega$$  
   (2)

2. $$\psi_\xi - \sum_{\xi \in \Omega} p_\xi \psi_\xi + \theta_\xi \geq 0, \quad \forall \xi \in \Omega$$  
   (3)

3. $$\sum_{j \in \Omega} x_{ij} \geq m_i y_i + (1 - y_i), \quad \forall i \in V$$   
   (4)

4. $$\sum_{j \in \Omega} x_{ij} \leq M_i y_i + (1 - y_i), \quad \forall i \in V$$  
   (5)
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\[
\sum_{i \in \mathcal{V}} x_{ij} \geq m_{ij} y_{j} + (1 - y_{j}), \quad \forall j \in \mathcal{V} 
\]

\[
\sum_{i \in \mathcal{V}} x_{ij} \leq M_{ij} y_{j} + (1 - y_{j}), \quad \forall j \in \mathcal{V} 
\]

\[
u_{i} + (L - 2) \sum_{k \in \mathcal{F}} x_{ik} y_{k} - \sum_{j \in \mathcal{V}} x_{ij} y_{j} \leq L - 1 + (1 - y_{j}) \beta, \quad \forall i \in \mathcal{V} 
\]

\[
u_{i} + \sum_{k \in \mathcal{F}} x_{ik} y_{k} + (2 - K) \sum_{j \in \mathcal{V}} x_{ij} y_{j} \geq 2(1 - y_{j}), \quad \forall i \in \mathcal{V} 
\]

\[
y_{i}(1 - y_{j})(x_{ij} + x_{ji}) \leq 1, \quad \forall i, j \in \mathcal{V} 
\]

\[
u_{i} - u_{ij} + Lx_{ij} + (L - 2)x_{ji} \leq L - 1 + (y_{i} + y_{j}) \beta, \quad \forall i, j \in \mathcal{V} 
\]

\[
x_{ij} \leq (1 - y_{i}) + (1 - y_{j}), \quad \forall i, j \in \mathcal{V} 
\]

\[
\theta_{\xi} \geq 0, \quad \forall \xi \in \Omega 
\]

\[
u_{i}, y_{j}, x_{ij} \in [0, 1], \quad \forall i, j \in \mathcal{V} 
\]

In the above model, equation (1) is the objective function where \(f_{i}\) is the fixed cost of opening a depot in node \(i\). Considering the definition of \(y_{i}\), the first statement of the objective function calculates the total facility location cost. In equation (1), \(\psi_{\xi}\) represents the total routing cost under each scenario \(\xi \in \Omega\) with \(\Omega\) being a finite set of scenarios. Each scenario, \(\xi\), may occur with a probability of \(p_{\xi}\). Therefore, by introducing \(\lambda\) as the penalty for solution variance, the second part of the objective function calculates the routing costs under uncertainty. Together with constraints (2), (3) and (13), the second part of the objective function provides the robustness of the model.

In contrary to the fixed location costs, routing costs are not single payments and are incurred for several future periods of time. Hence, considering time value of money is essential for obtaining realistic results. Assuming the routing costs as uniform series payments and utilising the uniform series payments factor, the time value of money is incorporated into calculations to obtain the present worth of future routing costs. The uniform series payments factor can be written as:

\[
(P / A; r; n) = \frac{(1+r)^{n} - 1}{r(1+r)^{n}} 
\]

where \(n\) is the number of planning periods and \(r\) is the interest rate (White et al., 1983). According to assumptions, the network is expected to operate for, theoretically, infinite number of periods, i.e., \(n \to \infty\) and therefore we have:

\[
(P / A; r; \infty) = \frac{1}{r} 
\]

The term \(1 / r\) is used as a coefficient in equation (1) for converting uniform series payments of the routing costs to their present worth. Through this conversion, the present
worth of future routing costs may be added to the single payment of fixed facility location costs.

Constraint (2) defines \( \psi_{\xi} \) as the routing cost under scenario \( \xi \). As proposed by Yu and Li (2000), constraint (3) is used to linearise the original robust optimisation problem. Constraints (4) and (5) determine the minimum and the maximum number of salesmen departing from the depot (if opened) in each node. More specifically, if a depot is opened in node \( i \), then \( y_i = 1 \) and therefore, for \( \sum_{j \in \mathcal{V}} x_{ij} \), as the total number of salesmen departing from node \( i \), we have:

\[
m_i y_i \leq \sum_{j \in \mathcal{V}} x_{ij} \leq M_i y_i \tag{17}
\]

where \( m_i \) and \( M_i \) are the minimum and the maximum number of salesmen in a depot (if opened) in node \( i \), respectively. On the other hand if \( y_i = 0 \), then node \( i \) is an ordinary customer node (i.e., with no depots) and from constraints (4) and (5) we have:

\[
\sum_{j \in \mathcal{V}} x_{ij} = 1 \tag{18}
\]

Equation (18) indicates that exactly 1 salesman must leave any customer node \( i \). Similarly, constraints (6) and (7) impose restrictions on the number of salesmen arriving at each node \( i \).

Considering \( u_i \) as the visit number of the node \( i \) and \( B \) as a sufficiently large positive number, constraints (8) and (9) confine the number of nodes visited on any salesman’s path from the depot up to node \( i \) to lie between a lower bound, \( K \), and an upper bound, \( L \). In addition, if a node \( i \) is the first node of a tour, these constraints translate to \( u_i = 1 \) initialising the value for this variable.

Constraint (10) prohibits the salesmen to visit only a single node. Assuming that \( B \) is the same as in constraint (8), constraint (11), known as sub-tour elimination constraint (SEC), does not allow for forming of sub-tours among customer nodes. To ensure that no salesman travels from a depot to another, constraint (12) is added to the model as an extra constraint. Finally, constraints (13) and (14) determine the types of the variables.

### 2.3 Linearisation

The mathematical model, described in the previous section, is obviously non-linear because of the multiplication of binary variables in constraints (8), (9) and (10). In order to convert the non-linear model to a linear one, an auxiliary binary variable is defined to replace the multiplication of binary variables. More specifically, let \( x \) and \( y \) are two binary variables multiplied in the model. By defining the auxiliary variable as \( z = \prod_{i=1}^{n} x_i \), replacing the term \( \prod_{i=1}^{n} x_i = x_1 \times x_2 \times \ldots \times x_n \) by \( z \) in the model and adding the constraint, the model can be linearised.

\[
\delta z \leq \sum_{i=1}^{n} x_i \leq z + \delta \tag{19}
\]
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In the above constraint, \( \delta \) is an arbitrary real number satisfying \( n - 1 < \delta < n \). Considering the definition of the auxiliary variable and constraint (19), the non-linear constraints (8), (9) and (10) are replaced with the following linear ones assuming \( z^1_{ij} = y_i(1 - y_j)x_{ij} \), \( z^2_{ij} = y_i(1 - y_j)x_{ij} \), \( z^3_{ij} = y_jx_{ij} \), \( z^4_{ij} = y_jx_{ij} \), \( 2 < \delta_1 < 3 \), \( 2 < \delta_2 < 3 \), \( 1 < \delta_3 < 2 \) and \( 1 < \delta_4 < 2 \).

\[
\begin{align*}
\sum_{k \in V} z^1_{ij} - \sum_{j \in V} z^2_{ij} & \leq L - 1 + (1 - y_i)B, & \forall i \in V \\
\sum_{k \in V} z^1_{ij} + (2 - K)\sum_{j \in V} z^4_{ij} & \geq 2(1 - y_i), & \forall i \in V \\
z^1_{ij} + z^2_{ij} & \leq 1, & \forall i, j \in V \\
z^3_{ij} - y_j - x_{ij} + \delta_3 & \geq 0, & \forall i, j \in V \\
\delta_1 z^3_{ij} - y_j - x_{ij} & \leq 0, & \forall i, j \in V \\
z^4_{ij} - y_j - x_{ij} + \delta_4 & \geq 0, & \forall i, j \in V \\
\delta_2 z^4_{ij} - y_j - x_{ij} & \leq 0, & \forall i, j \in V \\
z^1_{ij} - y_i - (1 - y_j) - x_{ij} + \delta_1 & \geq 0, & \forall i, j \in V \\
\delta_1 z^1_{ij} - y_i - (1 - y_j) - x_{ij} & \leq 0, & \forall i, j \in V \\
z^3_{ij} - y_i - (1 - y_j) - x_{ij} + \delta_3 & \geq 0, & \forall i, j \in V \\
\delta_2 z^3_{ij} - y_i - (1 - y_j) - x_{ij} & \leq 0, & \forall i, j \in V
\end{align*}
\]

\( 3 \) Computational results

In order to study the performance of the proposed model and to justify its applicability, a central Iran case of dairy supply chain and several numerical examples are solved by using the integer linear programme (ILP) solver of LINGO 9.0 Software. For our experiments, we utilised a desktop Personal Computer (PC) equipped with an Intel® Core™ i3 3210 @ 2.30 GHz CPU and 4GB of RAM running Microsoft Windows® 7 Ultimate™.

\( 3.1 \) The dairy supply chain case study

The increase in urbanisation, population and income is expected to result in a growth in demand for livestock food products (Delgado et al., 1999). In Iran, the production and consumption of dairy products are expected to rise in future (OECD/FAO, 2012). Researchers have already been attracted to dairy supply chains in Iran (Khalili-Damghani et al., 2012). The consumption of these products in Iran is increasing
every year not only because of the population growth and welfare improvements but also because of the government health and food programmes (Nambiar et al., 1989). The increase in demand creates new markets, e.g., in Ethiopia emerging markets in Addis Ababa has attracted many rural producers (Francesconi et al., 2010). In order to have a share of the market, producers and distributors should make SCM decisions including the routing and location planning.

The geographic region of our study is the central Iran, where the industrial growth and urbanisation rate is relatively high, is presented. More specifically, in Markazi province in this region, the total investment, estimated based on the total number of patents issued by Industries and Mines Organization, grew from 21.68 million USD in 2001 to 622.04 million USD in 2011 demonstrating a 270% increase (Iran Statistics Center, 2012a). Furthermore, the urban population of the area of study has had an average growth of 5.6% and 2.8% from 2006 to 2011, respectively (Iran Statistics Center, 2012b, 2012c).

The cities in the region under investigation are Ashtiyan, Arak, Delijan, Golpayegan, Qom, Khomeyn, Tafresh and Mahallat are depicted in Figure 2. The distance between each two of these cities is provided in Table 2. The uncertain routing costs are calculated based on these data and the fixed facility investment cost coefficient for each node is obtained from feasibility studies provided by the Ministry of Cooperatives (Bank Loans and Economic Affairs Office, 2006).

**Figure 2** The map view of the area under investigation (see online version for colours)

Source: Gitashenasi (2009)
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Table 2: The distance between each two of the cities in the study

<table>
<thead>
<tr>
<th>Distance (KM)</th>
<th>Mahallat</th>
<th>Qom</th>
<th>Arak</th>
<th>Delijan</th>
<th>Khomeyn</th>
<th>Golpayegan</th>
<th>Tafresh</th>
<th>Ashtiyan</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mahallat</td>
<td>0</td>
<td>125</td>
<td>120</td>
<td>25</td>
<td>45</td>
<td>75</td>
<td>181</td>
<td>160</td>
</tr>
<tr>
<td>Qom</td>
<td>125</td>
<td>0</td>
<td>120</td>
<td>100</td>
<td>160</td>
<td>195</td>
<td>120</td>
<td>100</td>
</tr>
<tr>
<td>Arak</td>
<td>120</td>
<td>120</td>
<td>0</td>
<td>145</td>
<td>80</td>
<td>110</td>
<td>95</td>
<td>75</td>
</tr>
<tr>
<td>Delijan</td>
<td>25</td>
<td>100</td>
<td>145</td>
<td>0</td>
<td>65</td>
<td>95</td>
<td>140</td>
<td>120</td>
</tr>
<tr>
<td>Khomeyn</td>
<td>45</td>
<td>160</td>
<td>80</td>
<td>65</td>
<td>0</td>
<td>35</td>
<td>220</td>
<td>240</td>
</tr>
<tr>
<td>Golpayegan</td>
<td>75</td>
<td>195</td>
<td>110</td>
<td>95</td>
<td>35</td>
<td>0</td>
<td>250</td>
<td>270</td>
</tr>
<tr>
<td>Tafresh</td>
<td>181</td>
<td>120</td>
<td>95</td>
<td>140</td>
<td>220</td>
<td>250</td>
<td>0</td>
<td>20</td>
</tr>
<tr>
<td>Ashtiyan</td>
<td>160</td>
<td>100</td>
<td>75</td>
<td>120</td>
<td>240</td>
<td>270</td>
<td>20</td>
<td>0</td>
</tr>
</tbody>
</table>

Three different scenarios are considered in this study: the first scenario illustrates a moderate situation in which the routing costs have their normal expected values. In the second and third scenarios, these parameters are 10% and 20% greater than those of the first scenario, respectively. The scenarios have probabilities of 0.5, 0.3 and 0.2, respectively. The interest rate is set to 20% based on estimation from a field study of Iranian financial markets and banks.

The minimum and maximum number of vehicles (salesmen) in each depot is set to 1 and 3, respectively. The fleet of homogenous vehicles should transport dairy products from the depots to the customers on their routes. The vehicles may return to either their original depot or a depot other than the original one. The minimum and the maximum tour length are 2 and 4, respectively. The coefficient, $\lambda$, is set to 10 which translates to a 10 USD penalty coefficient for deviations.

With the aforementioned parameters, the linearised model of the problem is solved in 2 seconds. The optimal solution suggests opening the depots in three cities: Mahallat, Khomeyn and Ashtiyan. The optimal routes are presented in Table 3. From the results, it can be inferred that a single vehicle departs from the depot opened in Mahallat, and after visiting the customer node Delijan, it returns to its original departure node, Mahallat. Similarly, a single vehicle leaves the depot in Khomeyn, visits Golpayegan and returns to Khomeyn. Since two vehicles depart from Ashtiyan depot, the situation is slightly different. More specifically, one vehicle visits Arak and Qom and then returns to its original depot and the other leaves the depot, visits Tafresh and returns to Ashtiyan. It can be seen that for the four vehicles, the minimum and the maximum length of the tours are 2 and 3, respectively.

Table 3: Optimal routes for the case study

<table>
<thead>
<tr>
<th>Route #</th>
<th>Depots</th>
<th>Second city</th>
<th>Third city</th>
<th>Fourth city</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Mahallat</td>
<td>Delijan</td>
<td>Mahallat</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Khomeyn</td>
<td>Golpayegan</td>
<td>Khomeyn</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Ashtiyan</td>
<td>Arak</td>
<td>Qom</td>
<td>Ashtiyan</td>
</tr>
<tr>
<td>4</td>
<td>Ashtiyan</td>
<td>Tafresh</td>
<td>Ashtiyan</td>
<td></td>
</tr>
</tbody>
</table>
3.2 Numerical examples

In order to further study the performance of the proposed model, several test problems are solved under the same conditions mentioned in the previous section. The test problems are categorised in three groups according to their size: small, medium and large. Each group is divided to levels according to the number of nodes and number of scenarios in problems and each level is comprised of 5 randomly generated problems of the same size. More specifically, in our experiments there are 5, 6 and 5 problem levels in small, medium and large groups, respectively. In the Small group, for example, P02 represents a set of 5 problems with 5 scenarios and 7 nodes. Therefore, a total of 80 problems are solved in our experiments and the average computational times for obtaining the corresponding optimal solutions are reported in Table 4.

<table>
<thead>
<tr>
<th>Size</th>
<th>Problem</th>
<th>Number of scenarios</th>
<th>Number of nodes</th>
<th>Time (h:mm:ss)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small</td>
<td>P01</td>
<td>3</td>
<td>6</td>
<td>0:00:01</td>
</tr>
<tr>
<td></td>
<td>P02</td>
<td>5</td>
<td>7</td>
<td>0:00:02</td>
</tr>
<tr>
<td></td>
<td>P03</td>
<td>3</td>
<td>8</td>
<td>0:00:03</td>
</tr>
<tr>
<td></td>
<td>P04</td>
<td>4</td>
<td>9</td>
<td>0:00:15</td>
</tr>
<tr>
<td></td>
<td>P05</td>
<td>5</td>
<td>10</td>
<td>0:00:17</td>
</tr>
<tr>
<td>Medium</td>
<td>P06</td>
<td>3</td>
<td>12</td>
<td>0:03:34</td>
</tr>
<tr>
<td></td>
<td>P07</td>
<td>2</td>
<td>15</td>
<td>0:05:47</td>
</tr>
<tr>
<td></td>
<td>P08</td>
<td>3</td>
<td>14</td>
<td>0:09:11</td>
</tr>
<tr>
<td></td>
<td>P09</td>
<td>4</td>
<td>15</td>
<td>0:10:50</td>
</tr>
<tr>
<td></td>
<td>P10</td>
<td>5</td>
<td>16</td>
<td>0:21:50</td>
</tr>
<tr>
<td></td>
<td>P11</td>
<td>4</td>
<td>17</td>
<td>0:40:12</td>
</tr>
<tr>
<td>Large</td>
<td>P12</td>
<td>4</td>
<td>18</td>
<td>1:07:43</td>
</tr>
<tr>
<td></td>
<td>P13</td>
<td>6</td>
<td>17</td>
<td>1:43:17</td>
</tr>
<tr>
<td></td>
<td>P14</td>
<td>5</td>
<td>18</td>
<td>2:25:02</td>
</tr>
<tr>
<td></td>
<td>P15</td>
<td>3</td>
<td>19</td>
<td>3:32:37</td>
</tr>
<tr>
<td></td>
<td>P16</td>
<td>4</td>
<td>20</td>
<td>4:37:10</td>
</tr>
</tbody>
</table>

Furthermore, Figure 3 depicts the curve of the average computational times for the levels. As can be seen, the time spent on solving the problem increases as the size of the problem grows. Therefore, for large scale problems, an efficient meta-heuristic solution method should be developed. However, since many real world cases for small and medium sized enterprises (SMEs) are of small or medium size, the corresponding linearised proposed model can be solved in a reasonable amount of time.
4 Conclusions and future works

In this paper, a robust location-routing problem (RLRP) model based on multi-depot multi-travelling salesmen problem (MmTSP) was presented. The uncertainty in transportation costs was modelled through stochastic parameters whose values were determined under scenarios (Mulvey et al., 1995). The applicability of the model was shown through a central Iran case study of dairy supply chain. The proposed model provided a general approach to RLRP; therefore, it may be utilised for incorporating location and routing decisions in many real world applications and problems [e.g., bus routing problem (BRP)].

In addition, the experiments with numerical examples of different sizes showed that the proposed model can be solved in a reasonable amount of time for small- and medium-sized problems using conventional solution methods such as B&B. Since many RLRPs in SMEs are of small and medium size, the proposed linearised model is applicable. Furthermore, location and routing are strategic and tactical decision, respectively, and are considered as design decisions which are not made very frequently. Therefore, the computational time for the proposed model is justifiable.

Nevertheless, developing an efficient meta-heuristic method for large-scale problems may be a subject of future research. Studying a commodity-flow formulation of RLRP considering uncertainty in other parameters such as product demand may also be an interesting field of future study. In addition, modelling other types of uncertainty, such as fuzziness or stochastic fuzziness, for the parameters also remains for future works.
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